Burger

Test 3 solutions

M2 6

1.

$$= \int \frac{\ln(2x)}{x^2}\,dx$$

Int. by Parts!

let $u = \ln 2x \longrightarrow du = \frac{1}{2x}\cdot 2 = \boxed{\frac{1}{x}\,dx = du}$

$\boxed{dv = \frac{1}{x^2}\,dx} \longrightarrow v = \int x^{-2}\,dx$

$\boxed{v = -\frac{1}{x}}$

$$= uv - \int v\,du$$

$$= \frac{-(\ln 2x)}{x} - \int \left(-\frac{1}{x}\right)\cdot \frac{1}{x}\,dx$$

$$= \frac{-\ln 2x}{x} + \int \frac{1}{x^2}\,dx$$

$$= \boxed{\frac{-\ln 2x}{x} - \frac{1}{x} + C}$$

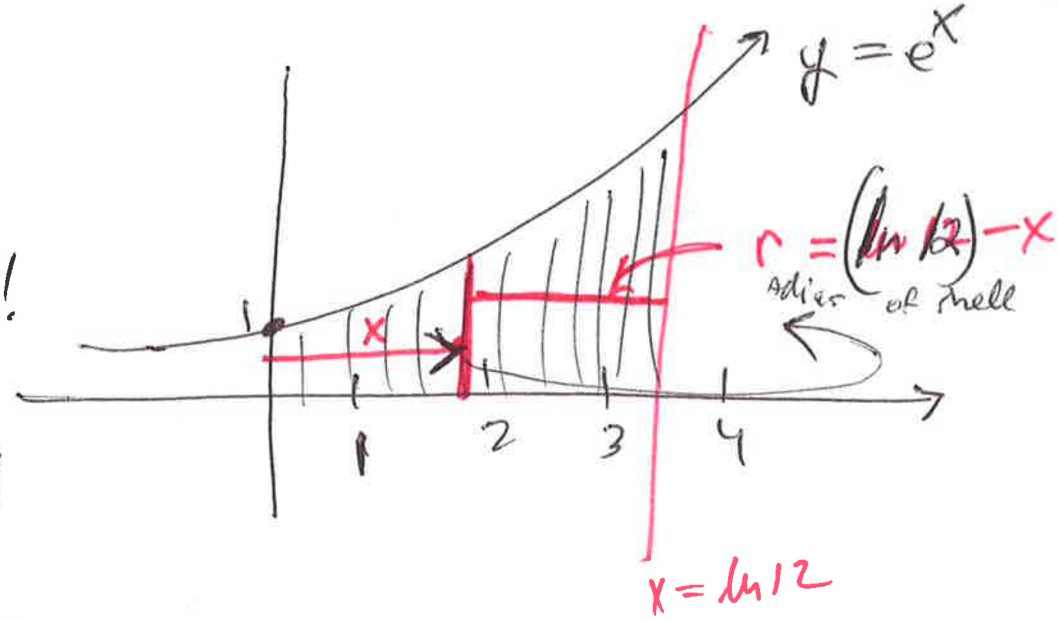
A

#2

$\ln 12 \approx$ 3 ish!

2.7

$(2.7)^3 < 12 < (2.7)^4$



Shells

$$V = 2\pi \int_0^{\ln 12} (\ln 12 - x) \cdot e^x \, dx \Bigg\} \text{ BASIC set-up}$$

$$V = 2\pi \underbrace{\int_0^{\ln 12} \ln 12 \cdot e^x \, dx}_{\text{easy!}} - 2\pi \int_0^{\ln 12} x e^x \, dx$$

By parts

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$e^{\ln 12} = 12$

$$\boxed{2\pi \ln 12 \, e^x \Big|_0^{\ln 12}}$$

$e^{\ln 12} = 12$

$2\pi \ln 12 \cdot 12 - 2\pi \ln 12 \cdot 1$

$24\pi \ln 12 - 2\pi \ln 12$

$22\pi \ln 12$

1st piece!

$= xe^x - \int e^x dx$

$xe^x - e^x \Big|_0^{\ln 12}$

$(\ln 12)12 - 12 -$

$0 - 12$

$\cancel{12 \ln 12 - 12 + 12} = 12 \ln 12$

2nd piece without -2π

Answer $V = 22\pi \ln 12 - 2\pi(12 \ln 12)$

$2\pi(11$

$2\pi \ln 12 \cdot 12 - 2\pi \ln 12 \cdot 1$

$24\pi \ln 12 - 2\pi \ln 12$

$22\pi \ln 12$

$$\int_0^{\ln 12} x e^x dx =$$

$$x e^x - \int e^x dx \Big|_0^{\ln 12} =$$

$$x e^x - e^x \Big|_0^{\ln 12} =$$

$$[\ln 12 \cdot 12 - 12] - [0 - 1]$$

$$\boxed{12 \ln 12 - 11}$$

$\hookrightarrow -2\pi(12\ln 12 - 11) =$

$\cancel{2\pi}$ $-24\pi \ln 12 + 22\pi$

$22\pi \ln 12 - 24\pi \ln 12 + 22\pi =$

$\cancel{22\pi - 2\pi \ln 12}$

$= \boxed{2\pi(11 - \ln 12)}$ D

#2 contd.

#3 $= \int_0^1 \frac{dx}{\sqrt{25-x^2}}$

let $x = 5u$

$dx = 5\,du$

$u = \frac{x}{5}$

$u(1) = \frac{1}{5}$

$u(0) = 0$

$$= \int_{u=0}^{\frac{1}{5}} \frac{\cancel{5}\,du}{\cancel{5}\sqrt{1-u^2}}$$

$$= \int_0^{\frac{1}{5}} \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u)\Big|_0^{1/5}$$

$$= \sin^{-1}\left(\tfrac{1}{5}\right) - \sin^{-1}(0)$$

$$= \boxed{\sin^{-1}\left(\tfrac{1}{5}\right)}$$

D

#4

$$= \int \frac{dx}{x\sqrt{64x^2-9}}$$

$$= \int \frac{dx}{x\sqrt{9\left(\left(\frac{8}{3}x\right)^2-1\right)}}$$

let

$\frac{8}{3}x = \sec\theta$

$x = \frac{3}{8}\sec\theta$

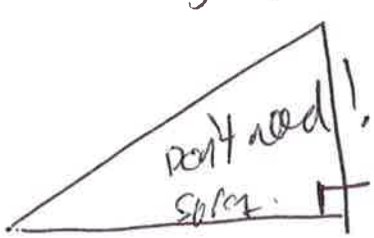
$dx = \frac{3}{8}\sec\theta\tan\theta\, d\theta$

$$= \frac{3}{8}\int \frac{\sec\theta\tan\theta\, d\theta}{\frac{3}{8}\sec\theta\sqrt{9(\tan^2\theta)}}$$

$$= \frac{3}{8}\cdot\frac{8}{3}\cdot\frac{1}{3}\int \frac{\cancel{\sec\theta}\,\cancel{\tan\theta}\, d\theta}{\cancel{\sec\theta}\,\cancel{\tan\theta}}$$

$$= \frac{1}{3}\int d\theta = \frac{1}{3}\theta + C$$

$\sec\theta = \frac{8}{3}x \longrightarrow \theta = \sec^{-1}\left(\frac{8}{3}x\right)$



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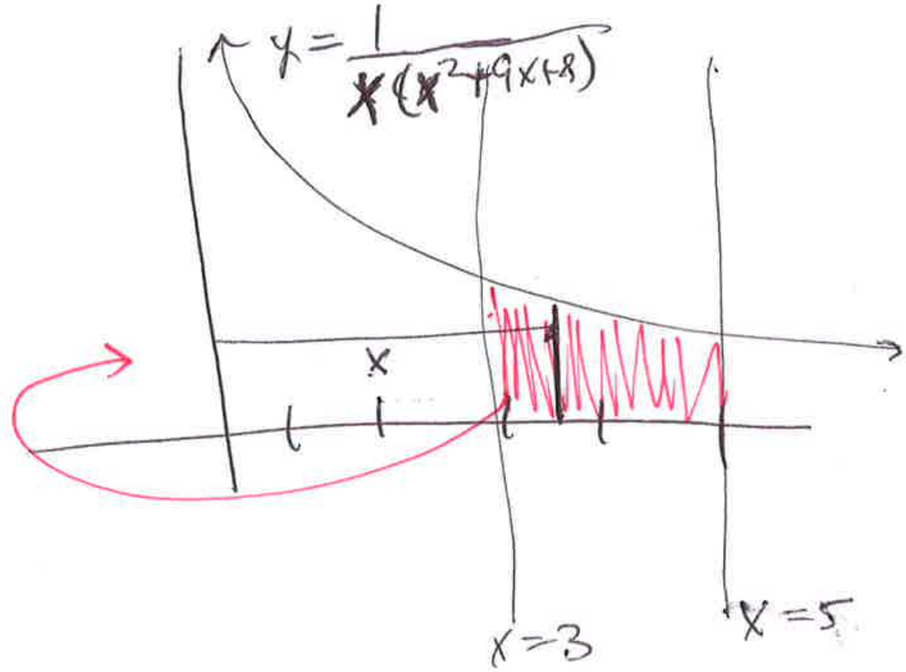
$$= \boxed{\frac{1}{3}\sec^{-1}\left(\frac{8}{3}x\right) + C}$$

#5



By Shells

$$V = 2\pi \int_3^5 \cancel{x} \cdot \frac{1}{\cancel{x}(x+1)(x+8)}\, dx$$

$$= 2\pi \int_3^5 \frac{1}{(x+1)(x+8)}\, dx$$

Partial Fractions

$$\frac{1}{(x+1)(x+8)} = \frac{A}{x+1} + \frac{B}{x+8}$$

$$Ax+8A+Bx+B=1$$
$$(A+B)x + (8A+B) = 1$$
$$A+B=0$$
$$8A+B=1$$
$$7A=1 \qquad \boxed{A=\frac{1}{7} \quad B=-\frac{1}{7}}$$

$$= 2\pi \int_3^5 \frac{1/7}{x+1}\, dx + 2\pi \int_3^5 \frac{-1/7}{x+8}\, dx$$

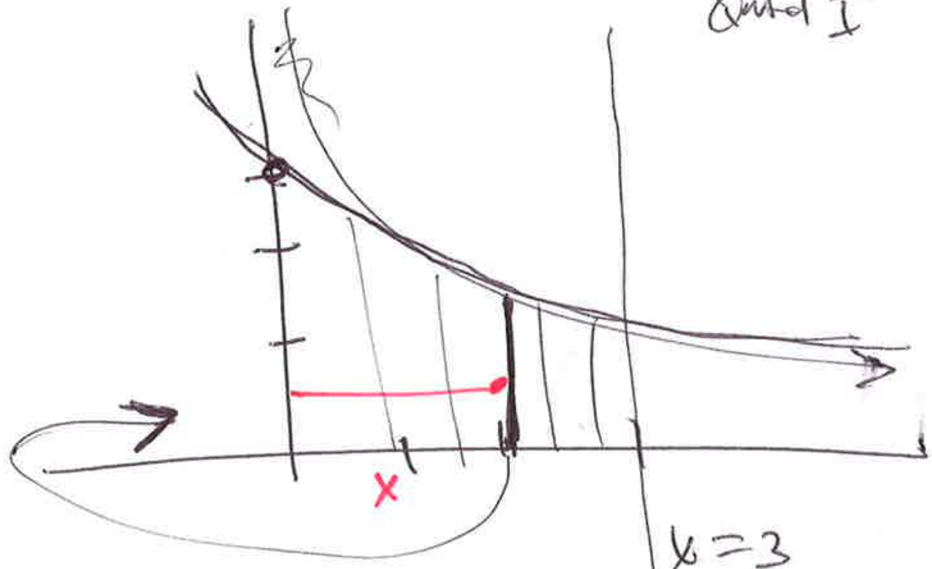
$$= \frac{2\pi}{7}\left(\ln|x+1| - \ln|x+8|\right)\Big|_3^5$$

$$\frac{2\pi}{7}\left(\ln 6 - \ln 13 - \ln 4 + \ln 11\right) = \ln\frac{6}{13\cdot 4} + \ln 11 = \ln\frac{66}{52} = \boxed{\ln\frac{33}{26}}$$

B

$$\ln\frac{6}{13} - \ln 4 = \ln\frac{6}{13\cdot 4}$$

#6



By shells:

$$V = 2\pi \int_0^3 x \cdot \frac{10}{x^4+4x^2+3}\,dx = 20\pi \int_0^3 \frac{x}{(x^2+1)(x^2+3)}\,dx$$

$x^4+4x^2+3 = (x^2+1)(x^2+3)$ Both irreducible!

$$\frac{x}{(x^2+1)(x^2+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+3}$$

$$\rightsquigarrow (Ax+B)(x^2+3) + (Cx+D)(x^2+1) = x$$

$$Ax^3 + 3Ax + Bx^2 + 3B + Cx^3 + Dx^2 + Cx + D = x$$

$\boxed{A+C=0}$ $\boxed{3A+C=1}$ $\boxed{B+D=0}$ $\boxed{3B+D=0}$

$$\begin{aligned} 3A + C &= 1 \\ -\quad A + C &= 0 \\ \hline 2A &= 1 \end{aligned}$$

$A = 1/2$
$C = -\frac{1}{2}$

$D = 0$

$$\begin{aligned} 3B + D &= 0 \\ -\quad B + D &= 0 \\ \hline 2B &= 0 \end{aligned}$$

$B = 0$

$$\frac{x}{(x^2+1)(x^2+3)} = \frac{\frac{1}{2}x}{x^2+1} - \frac{1/2\,x}{x^2+3}$$

6 contd.

$$V = \frac{20\pi}{2}\int_0^3 \frac{x}{x^2+1}\,dx - 10\pi\int_0^3 \frac{x}{x^2+3}\,dx$$

$u = x^2+1$
$du = 2x\,dx$
$\frac{1}{2}du = dx$

$u = x^2+3$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$$\frac{1}{2}10\pi \ln|x^2+1|\Big|_0^3 - \frac{1}{2}10\pi \ln|x^2+3|\Big|_0^3$$

$$5\pi\left(\ln 10 - \ln\cancel{1} - \ln 12 + \ln 3\right)$$

$$= 5\pi\left(\ln\frac{10}{12} + \ln 3\right)$$

$$= 5\pi\left(\ln\frac{30}{12}\right) = 5\pi\ln(5/2)$$

A

Test 3

#7.

M76 Burger

$$I = \int \frac{dx}{(x^2-4)^{3/2}} \qquad x > 2$$

let $\boxed{x = 2\sec\theta} \rightarrow dx = 2\sec\theta\tan\theta\, d\theta$

$$I = \int \frac{2\sec\theta\tan\theta\, d\theta}{\left(\sqrt{4\sec^2\theta - 4}\right)^3}$$

$$= 2\int \frac{\sec\theta\tan\theta\, d\theta}{8\,(\underbrace{\sqrt{\sec^2\theta-1}}_{\tan^2\theta})^3} = \frac{1}{4}\int \frac{\sec\theta\tan\theta\, d\theta}{(\tan\theta)^3}$$

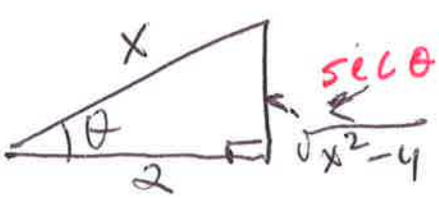
$$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta = \frac{1}{4}\int \frac{\cos^2\theta\, d\theta}{\cos\theta\sin^2\theta}$$

$$= \frac{1}{4}\int \frac{\cos\theta\, d\theta}{\sin^2\theta} = \frac{1}{4}\int u^{-2}\, du = \frac{u^{-1}}{-4} + C$$

let $u = \sin\theta$

$du = \cos\theta\, d\theta$

$$= -\frac{1}{4\sin\theta} + C$$



$\sec\theta = \frac{x}{2}$

$$= -\frac{1}{4\frac{\sqrt{x^2-4}}{x}} + C = \boxed{\frac{-x}{4\sqrt{x^2-4}} + C}$$

A

M76 Test 3 Burger

#8.

$$= \int \frac{dx}{(x^2+36)^{3/2}}$$

Let $\boxed{x = 6\tan\theta} \rightarrow dx = 6\sec^2\theta \, d\theta$

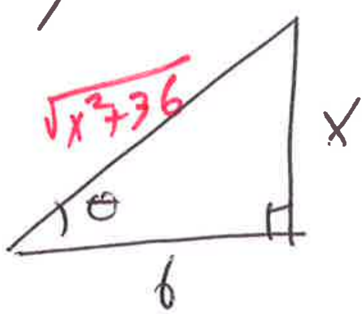
note: $\tan\theta = \frac{x}{6}$ ← (need this at the end!)

$$= 6\int \frac{\sec^2\theta \, d\theta}{\left(\sqrt{36\tan^2\theta+36}\right)^3} \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$= 6\int \frac{\sec^2\theta \quad d\theta}{6^3\left(\underbrace{\sqrt{\tan^2\theta+1}}_{\sec^2\theta}\right)^3}$$

$$= \frac{6}{6^3}\int \frac{\sec^2\theta \; d\theta}{\sec^3\theta} = \frac{1}{36}\cdot\int \frac{1}{\sec\theta}\, d\theta$$

$$= \frac{1}{36}\int \cos\theta \, d\theta = \frac{1}{36}\sin\theta + C$$



$$\therefore \boxed{\frac{1}{36}\cdot\frac{x}{\sqrt{x^2+36}} + C}$$

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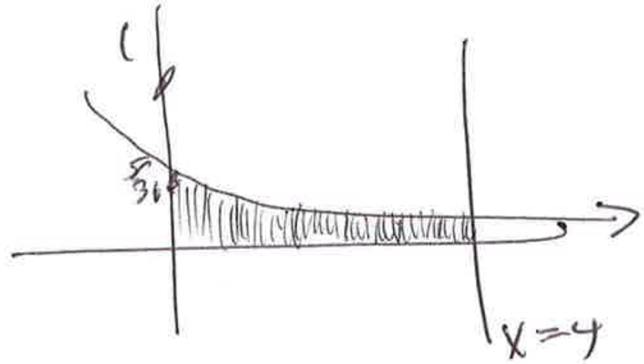
#9.



$$\text{Area} = 5\int_0^4 \frac{1}{36+25x^2}\,dx$$

let $u = \frac{5}{6}x$

$du = 5/6\,dx$

$\rightarrow dx = \frac{6}{5}\,du$

$$= 5\int_0^4 \frac{1}{36\left(1+\left(\frac{5x}{6}\right)^2\right)}$$

$$= \cancel{5}\cdot\frac{6}{\cancel{5}}\cdot\frac{1}{36}\int_{x=0}^{4} \frac{1}{1+u^2}\,du = \frac{1}{6}\tan^{-1}(5/6x)\Big|_0^4$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{20}{6}\right) - \frac{1}{6}\cdot 0$$

$$= \boxed{\frac{1}{6}\tan^{-1}\left(\frac{10}{3}\right)}$$

A

#10

$$\int_{-1}^{1} \frac{5}{1+25t^2}\,dt$$

let $u = 5t$

$du = 5\,dt$

$dt = \frac{1}{5}\,du$

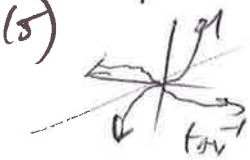
similar to last problem:

$$5\cdot\frac{1}{5}\int_{t=-1}^{1} \frac{1}{1+u^2}\,du = \tan^{-1}(5t)\Big|_{-1}^{1}$$

$$= \tan^{-1}(5) - \tan^{-1}(-5)$$

$$- -\tan^{-1}(5)$$

odd function



$$= \tan^{-1}(5) + \tan^{-1}(5)$$

$$= \boxed{2\tan^{-1}(5)}$$

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